



SRI VASAVI ENGINEERING COLLEGE

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Unit wise Question Bank

Department of Electronics and Communication Engineering

Academic year: 2017-18

Year/Semester: II/I

Programme: B Tech

Branch: ECE

Section: A, B & C

Course Code: R1621045

Course Title: Random Variables & Stochastic Processes

UNIT 1

1. Define and state the properties of (a) Distribution function and (b) Density function
2. Explain Continuous random variable functions
3. Define Random variable and give the concept of random variable.
4. In an experiment of rolling a die and flipping a coin. The random variable(X) is chosen such that: (i). A coin head (H) outcome corresponds to positive values of X that are equal to the numbers that show upon the die and (ii). A coin tail (T) outcome corresponds to negative values of X that are equal in magnitude to twice the number that shows on die. Map the elements of random variable X into points on the real line and explain.
5. In experiment where the pointer on a wheel of chance is spun. The possible outcomes are the numbers from 0 to 12 marked on the wheel. The sample space consists of the umbers in the set $\{0 < S < = 12\}$ and if the random variable X is defined as $X = X(S) = S^2$, map the elements of random variable on the real line and explain.
6. What are point conditioning and interval conditioning distribution function? Explain.
7. The random variable X has the discrete variable in the set $\{-1,-0.5, 0.7, 1.5, 3\}$ the corresponding probabilities are assumed to be $\{0.1, 0.2, 0.1, 0.4, 0.2\}$. Plot its distribution function and state is it a discrete or continuous distribution function.
8. A random variable X has the following probability function:

x: 0 1 2 3 4 5 6 7 8

P(x): a 3a 5a 7a 9a 11a 13a 15a 17a

- (i) Determine the value of 'a'. (ii) What is the smallest value of x for which $P(X \leq x) > 0.5$.(iii) Find $P(0 < x < 5)$.

UNIT 2

1. A discrete random variable x has possible values $x_i = i^2$, $i = 1, 2, 3, 4, 5$, which occur with probabilities $0.4, 0.25, 0.15, 0.1$, and 0.1 respectively. Find the mean value $x' = E[x]$ of x .
2. Given a random variable x and its density function
$$f_x(x) = 1, 0 < x < 1$$
$$= 0, \text{ otherwise}$$
Evaluate $x' = E[x]$
3. If x is a uniform random variable in the interval (x_1, x_2) . Find the expected value of x
4. Find the expected value of the function $g(x) = x^3$, where x is a random variable defined by the density function $f_x(x) = (1/2) \cdot u(x) \cdot \exp(-x/2)$.
5. Determine the mean value of the continuous exponentially distributed random variable with density function
$$f_x(x) = (1/b) \cdot \exp(-(x-a)/b); x > 0$$
$$= 0; x < 0$$
6. a) Explain the following terms (i). Conditional Expected value (ii). Covariance.
b) Find the Expected value of the number on a die when thrown.
7. Explain about moment generating function and characteristic function
8. Explain about transformations about a random variable.
9. State and prove properties of variance of a random variable.

UNIT 3

1. Write the properties of joint probability distribution and density function of two random variables.
2. Find the density function of $W = X + Y$ where the densities of X and Y are assumed to be $f_x(x) = (1/a) [u(x) - u(x-a)]$ and $f_y(y) = (1/b) [u(y) - u(y-b)]$; Where $0 < a < b$.
3. A joint probability density function is $f_{XY}(x,y) = (1/ab)$ for $0 < x < a$ and $0 < y < b$
$$= 0; \text{ else where.}$$
Find $F_{XY}(x,y)$
4. State and prove central limit theorem.
5. Find a constant 'b' so that the function $f_{XY}(x,y) = b \cdot e^{-(x+y)}$, $0 < x < a$ and $0 < y < \infty$
$$= 0, \text{ else where}$$
is a valid joint density function. (ii) Find an expression for the joint distribution function.

6. Discuss about statistical independence.
7. Given the function $f_{XY}(x,y) = (x^2 + y^2) / 8\pi$ $x^2 + y^2 < b$
 $= 0$ elsewhere
 - a) Find a constant 'b' so that this is a valid joint density function
 - b) Find $P(0.5b < X^2 + Y^2 \leq 0.8b)$
8. Given the function $f_{XY}(x,y) = b(x+y)^2$ $-2 < x < 2$ and $-3 < y < 3$
 $= 0$ elsewhere
 - a) Find a constant 'b' so that this is a valid joint density function
 - b) Find the marginal density functions $f_X(x)$ and $f_Y(y)$
9. Show that the variance of a weighted sum of uncorrelated random variables (weights α_i) equals the weighted sum of variances of the random variables (weights α_i^2)
10. Write the expression for expected value of a function of random variables and prove that the mean value of a weighted sum of random variables equals the weighted sum of mean values.
11. Random variables X and Y have the joint density
 $f_{XY}(x,y) = f(x) = (1/24)$, $0 < x < 6$ and $0 < y < 4$. What is the expected value of the function $g(x,y) = (x.y)^2$
12. The density function $f_{XY}(x,y) = f(x) = (x.y)/9$, $0 < x < 2$ and $0 < y < 3$ applies to two random variables. Show that x and y are uncorrelated.
13. X is a random variable with mean $E[X] = 3$, variance $\sigma^2_X = 2$.
 - i. Determine the second moment of X about origin
 - ii. Determine the mean of random variable y, where $y = -6X + 22$.
14. Two random variables x and y have the joint characteristic function
 $\Phi_{x,y}(w_1,w_2) = \exp(-2w_1^2 - 8w_2^2)$. Show that x and y are both zero-mean random variables and that they are uncorrelated.
15. State some of the properties exhibited by N jointly Gaussian random variables X_1, X_2, \dots, X_N

UNIT 4

1. Write the classification of random processes.
2. Show that the random process $x(t) = A \cos(\omega_0 t + \theta)$ is wide – sense stationary if it is assumed that A and ω_0 are constants and θ is a uniformly distributed random variable on the interval $(0, 2\pi)$.
3. What is mean- Ergodic process? Prove that the given $x(t)$ is a Mean – Ergodic process.

4. Define autocorrelation function and write its properties.
5. Define cross-correlation function of two random processes $x(t)$ and $y(t)$ write its Properties
6. Let $x(t)$ be a wide –sense stationary random process with auto correlation function $R_{xx}(\tau) = e^{-a|\tau|}$, Where $a > 0$ is a constant. We assume $x(t)$ “amplitude modulates” a “carrier” $\cos \omega_0 t + \theta$, where ω_0 is a constant and θ is a random variable uniform on $(-\pi, \pi)$ that is statistically independent of $x(t)$ Determine the autocorrelation function of $y(t)$.
7. Let two random processes $x(t)$ and $y(t)$ be defined by

$$X(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t)$$

$$Y(t) = B \cos(\omega_0 t) - A \sin(\omega_0 t)$$
 Where A and B are random variables and ω_0 is a constant. $x(t)$ is wide –sense stationary and, A and B are uncorrelated, zero –mean random variables with the same variance, $y(t)$ is wide –sense stationary. Show that $x(t)$ and $y(t)$ are jointly wide –sense stationary. Find the cross-correlation function $R_{xy}(t, t + \tau)$.
8. Discuss in detail about:
 - (a) First order stationary random process
 - (b) Second order & Wide - Sense Stationary Random Process.
9. (a) Prove that autocorrelation function of a random process is even function of τ .
 (b) Prove that $R_{xx}(\tau) = R_{xx}(0)$.

UNIT 5

1. A WSS random process $X(t)$ has $R_{xx}(\tau) = A_0[1 - (|\tau|/T)]$ $-T \leq \tau \leq T$
 $= 0$ elsewhere.

 Find power density spectrum.
2. $R_{xx}(\tau) = (A_0^2/2) \sin \omega_0 \tau$. Find $S_{xx}(\omega)$
3. Prove that PSD and auto correlation function of Random process form a Fourier transform pair.
4. A Random process has the power density spectrum $S_{xx}(\omega) = 6\omega^2/(1+\omega^4)$. Find the average power in the process.
5. Find the PSD of a random process $x(t)$ if $E[x(t)] = 1$ and $R_{xx}(\tau) = 1 + e^{-\alpha|\tau|}$.
6. Find the auto correlation function and power spectral density of the random process, $x(t) = k \cos(\omega_0 t + \theta)$, Where θ is a random variable over the ensemble and is uniformly distributed over the range $(0, 2\pi)$.

7. Define cross power density spectrum and state its properties

UNIT 6

1. a) What are the characteristics of White noise?
b) Discuss the spectral distribution of thermal noise.
2. a) Describe the behavior of zero mean stationary Gaussian band limited White noise.
b) The noise figure of an amplifier at room temperature ($T=290^0$ K) is 0.2dB, Find the equivalent temperature.
3. Derive the relation between PSD's of input and output random process of an LTI Systems.
4. Derive the equation of narrow band noise and illustrate its properties
5. (a) Two systems have transfer functions $H_1(\omega)$ & $H_2(\omega)$. Show the transfer function $H(\omega)$ of the cascade of the two is $H(\omega) = H_1(\omega) H_2(\omega)$.
(c) For cascade of N systems with transfer functions $H_n(\omega)$, $n=1,2,\dots,N$ show that $H(\omega) = \prod H_n(\omega)$.
6. What is the need for band limiting the signal towards the direction of increasing SVR?
7. What are the different noise sources that may be present in an electron device.
8. What are the precautions to be taken in cascading stages of a network from the point of view of noise reduction?